WF-Note 193
February 15, 2000

Measurement of Q for X-Band Dielectric Loaded Standing-Wave Accelerating Structure

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I. Introduction

The proposed use of RF driven dielectric based standing-wave structure for particle acceleration can be traced to the early 50's [1]. The motivation is to use modest RF power to achieve high gradient in a dielectric loaded resonant cavity. Due to the available RF source at Naval Research Laboratory (2-20MW at 11.424GHz), we would like to achieve >30MV/m gradient. The physical properties of a dielectric loaded standing-wave accelerating structure have been calculated [2]. Figure 1 shows the diagram of such structure. This note describes the measurement of quality factor of this dielectric loaded standing-wave cavity.

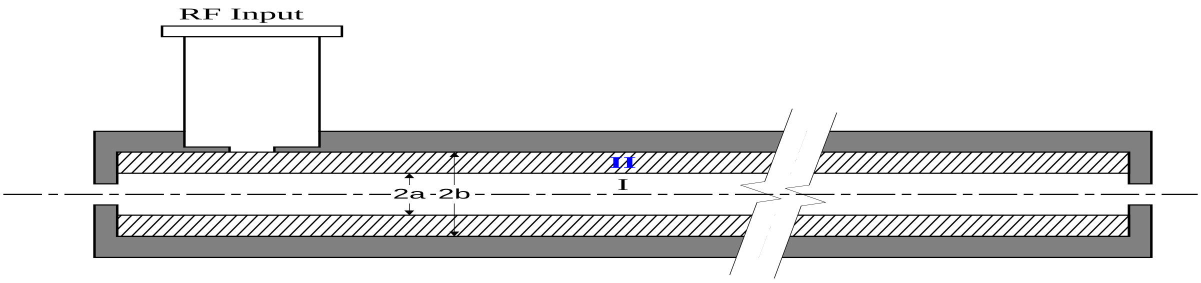
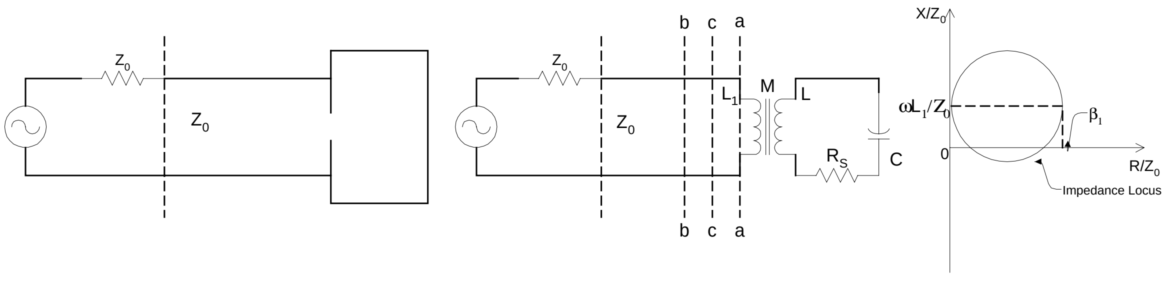


Figure 1 The Schematic Drawing of A Dielectric Loaded Standing-Wave Accelerator

II. Principles and Setup of Quality Factor of Dielectric Loaded Cavity

Figure 2.a shows a cavity coupled to a signal source whose internal impedance is equal to the characteristic impedance of the transmission line. Figure 2.b and c show the equivalent circuit and the impedance locus referred to the primary of the transformer. The coupling between the cavity and the transmission line is symbolically represented by an iris, which indicates some arbitrary method of exiting the cavity fields. The actual form of the coupling mechanism does not affect the equivalent circuit [3].



(a) **(b)** **(c)**

Figure 2 A Resonant Cavity and the Coupled Transmission

The impedance at the terminals of the coupling network a-a in figure 2(b) is equal to

$$Z_{aa} = jX_1 + \frac{(\omega M)^2}{R_s + j(\omega L - 1/\omega C)} \tag{1}$$

For high-Q cavities, the above equation can be written as

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$$\frac{Z_{aa}}{Z_0} = j\frac{X_1}{Z_0} + \frac{\beta_1}{1 + j2Q_0\delta}$$

$$\beta_1 = \frac{(\omega M)^2}{Z_0 R_s}$$

$$Q_0 = \frac{\omega L}{R_s} \qquad (2)$$

$$\delta = \frac{\omega - \omega_0}{\omega}$$

$$\omega_0^{\ 2} = 1/LC$$

The quantity β_1 is the ratio of the coupled resistance to the cavity resistance R_s, and it is also called as coupling coefficient. The quantity δ is called the frequency-tuning parameter. The second term of equation (2) corresponds to an equation of a circle in the rectangular impedance plane. This circle represents the impedance of a shunt resonance circuit with resonant impedance $\beta_1 Z_0$. The effect of the self-reactance of the coupling system is expressed by the first term, which is to displace the circle along the imaginary axis.

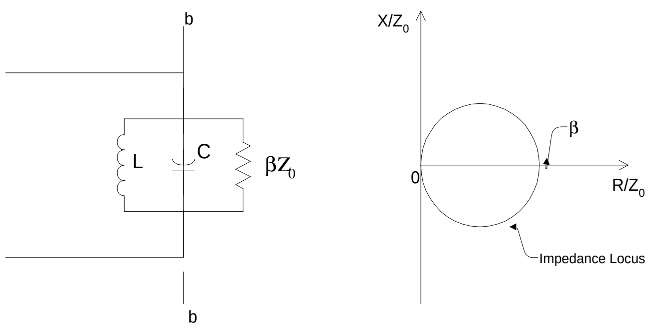
The impedance at the detuned short position, looking toward the cavity, has the form of a simple resonant shunt circuit shown in figure 3. If the terminals b-b is the detuned short position, and at a distance *l* away from the terminals a-a, the cavity impedance at a-a can be transformed to the terminals b-b, as

$$\frac{Z_{bb}}{Z_0} = \frac{Z_{aa} + jZ_0 \tan kl}{Z_0 + jZ_{aa} \tan kl} \qquad (3)$$

The quantity *k* is the propagation constant along the transmission line. If $\tan kl = -X_1/Z_0$, the impedance at the detuned short position for any value of δ becomes

$$\frac{Z_{bb}}{Z_0} = \frac{\beta}{1 + j2Q_0(\delta - \delta_0)}$$

$$\delta_0 = \frac{\beta}{2Q_0}\left(\frac{X_1}{Z_0}\right) \qquad (4)$$



(a) **(b)**

Figure 3 The Equivalent Circuit at the Detuned Short Position

The impedance loci in the impedance plane at terminals a-a and terminals b-b are shown in the figure 2(c) and 3(b), respectively. In the experimental setup in figure 4, the short pin is the coupling system between cavity and transmission line. This coupling scheme is to ensure the TM-like resonant mode excited. Its location corresponds to the terminals a-a in the figure 3. The reflection coefficients are measured at the reference plane c-c. Assume the electrical distance between reference plane a-a and c-c is l', the impedance locus corresponding to c-c is transformed from impedance locus of a-a according to equation (3). If the electrical distance between terminals c-c and b-b is l'', the simple impedance form at the detuned short position can be obtained by the same transformation as equation (3). It means we can certainly place the impedance locus at the location shown in figure 3(b) through the impedance transformation along the transmission line. That location in the transmission line corresponds to a detuned short position.

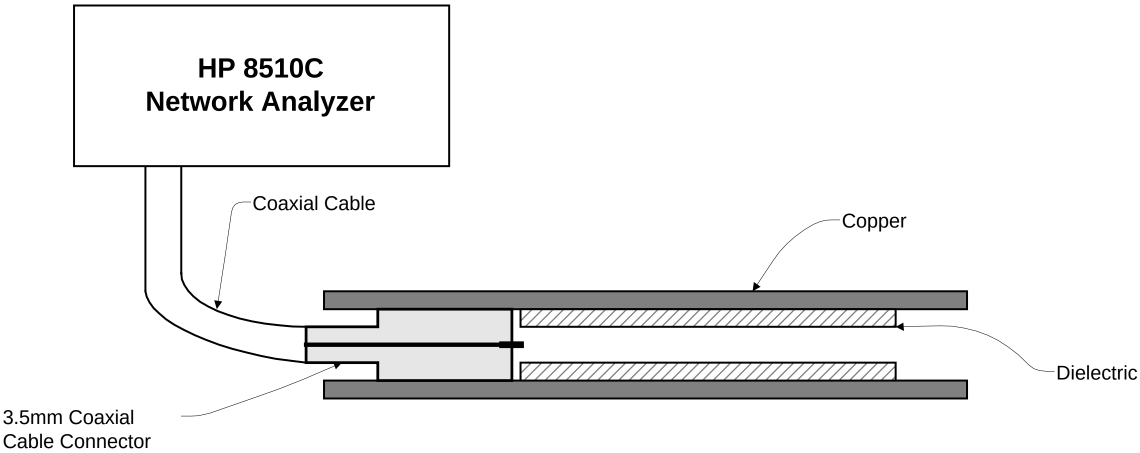


Figure 4 The Schematic Drawing of the Measurement Setup

As the measurement setup shown in figure 4, the dielectric loaded cavity is not terminated by two conducting planes. Perfect contact between these conducting planes and the wall of circular waveguide, and between the dielectric ceramic and these planes is very difficult to achieve. It causes more energy dissipation to the cavity and lower the quality factor. On the other hand, two ideal reflecting planes are assumed in the preliminary calculation [2]. Since the TM_{01} wave is under cut-off in the non-dielectric loaded waveguide part, the wave is totally reflected back to the dielectric loaded waveguide without loss. The gap between the coupling pin and dielectric ceramic is to ensure the good reflection.

If X and R are the imaginary and the real parts of the impedance given by equation (4), the ratios X/R for δ_p and δ_q are

$$\left(\frac{X}{R}\right)_q = -2Q_0(\delta_q - \delta_0)$$
$$\left(\frac{X}{R}\right)_p = -2Q_0(\delta_p - \delta_0) \tag{5}$$

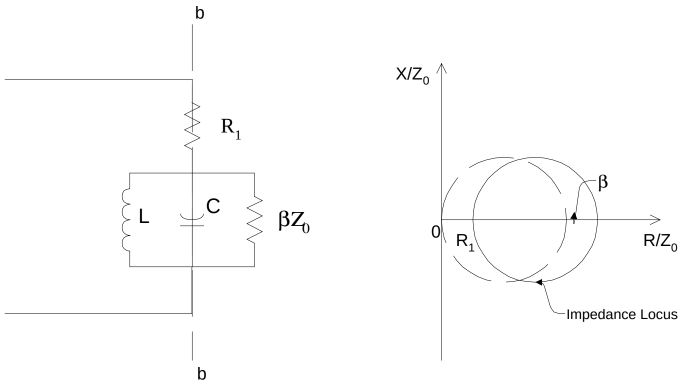
Subtracting and rearranging the above equations, the unloaded Q can be calculated as

$$Q_0 = \frac{1}{2}\frac{1}{\delta_p - \delta_q}\left[\left(\frac{X}{R}\right)_q - \left(\frac{X}{R}\right)_p\right] \tag{6}$$

Point p and q are the points on the impedance locus at the detuned short position. The impedance method for measuring the Q values is based on the assumption that the cavity's coupling network is lossless. If losses are present, the equivalent circuit is shown in figure 5. The loss in the coupling system can be represented as a resistor regardless of the actual cause of loss. If the self-reactance of the coupling system is sufficiently small, the impedance at terminal a-a and b-b are

$$\frac{Z_{aa}}{Z_0} = \frac{R_1 + jX_1}{Z_0} + \frac{\beta_1}{1 + j2Q_0\delta}$$
$$\frac{Z_{bb}}{Z_0} = \frac{R_1}{Z_0} + \frac{\beta_1}{1 + j2Q_0\delta} \tag{7}$$

The alteration of the impedance method removes the first term of the above equation. Thus, the Q values can be obtained from the impedance locus.



(a) **(b)**

Figure 5 The Equivalent Circuit at the Detuned Short Position with Loss Existing in the Coupling System

III. Measurement result of Q values

The complex reflection coefficients Γ are recorded from HP8510c network analyzer. The magnitude of reflection coefficients is shown in figure 6. And the impedance at the reference plane can be obtained from

$$\frac{Z_{cc}}{Z_0} = \frac{1+\Gamma}{1-\Gamma} \tag{8}$$

The impedance locus at the detuned short position can be found through the necessary transformation. The most left point of the impedance locus should be move to the location R=0, if losses are presented. In practice, the impedance locus may not be perfectly round, especially for Q value around several thousand. Averaging several sets of measurements gives more accurate result.

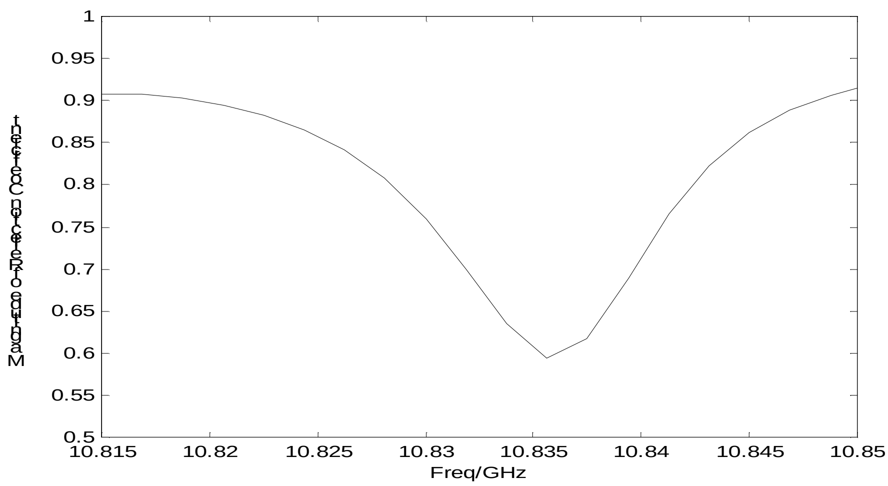


Figure 6 The Magnitude of the Reflection Coefficients of the First Resonance Mode

The impedance locus corresponding to the curve of figure 6 is shown in figure 7.

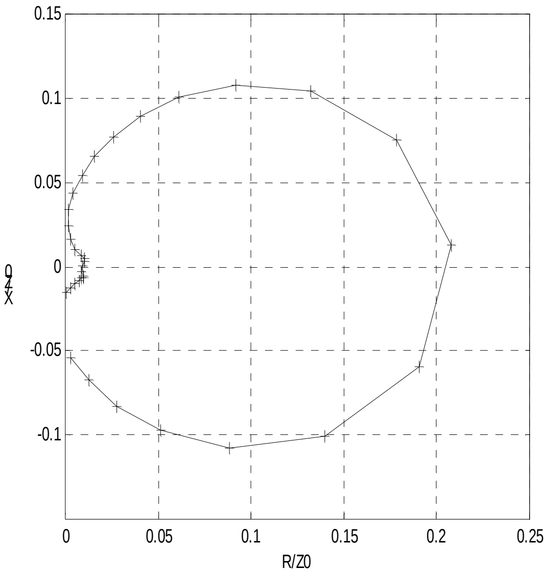


Figure 7 The Impedance Locus of the Measured Resonance Mode

The unloaded quality factor Q_0 obtained from this impedance locus are listed in the table I. The mean value of Q_0 is 1542.

Q Value of Set 1	Q Value of Set 2	Q Value of Set 3	Q Value of Set 4	Average
1555.3	2108.8	1318.2	1183.9	1541.55

Table I The Measurement Result of Q

The calculated value for this resonant mode is around 2100 [2]. A reasonable agreement between the measurement and calculation is achieved.

IV. Conclusion

The quality factor measurement for an X-band dielectric loaded standing-wave cavity is described. A prototype of such dielectric loaded X-band standing-wave accelerating structure will be constructed. The high power testing for this prototype will be conducted at the NLR in the next year. The experimental investigations on this prototype will provide more knowledge for the development of dielectric based accelerators.

Reference:

1. G. Flesher and G. Cohn, Dielectric Loading for Waveguide Linear Accelerators, AIEE Transactions, 70, 887-893,1951
2. X. Sun, Dispersion Relation and Quality Factor of TM01m Modes in Standing Wave Dielectric Structures, WF-Note 189, 1999
3. E. L. Ginzton, Microwave Measurements, Chapter 9, McGraw-Hill Book Company, Inc., 1957
4. Trans-Tech., Catalog, 5520 Adamstown, Adamstown, MD 21710